Greenberger-Horne-Zeilinger correlation and Bell-type inequality seen from moving frame

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Abstract

The relativistic version of the Greenberger-Horne-Zeilinger experiment with massive particles is proposed. We point out that, in the moving frame, GHZ correlations of spins in original directions transfer to different directions due to the Wigner rotation. Its effect on the degree of violation of Bell-type inequality is also discussed.

Key words: GHZ correlation, Bell-type inequality, relativity

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Contemporary applications of the Einstein, Podolsky, and Rosen (EPR) correlations and the Bell inequality range from purely theoretical problems [1,2,3] to quantum communication such as quantum teleportation [4] and quantum cryp-

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tography [5,6]. Recently a lot of interest has been devoted to the study of the EPR correlation function under the Lorentz transformations [7,8,9,10,11,12]. They showed that, the relativistic effects on the EPR correlations are nontrivial and the degree of violation of the Bell inequality depends on the relative motion of the particles and the observers.

Greenberger, Horne and Zeilinger (GHZ) also proposed a kind of quantum correlations known as GHZ correlations [13,14]. This kind of refutation of local realism is strikingly more powerful than the one Bell's theorem provides for Bohm's version of EPR — it is no longer statistical in principle. Furthermore, GHZ correlations are essential for most quantum communication schemes in practice [14,15,16,17]. Thus it is an interesting question that whether GHZ correlations can still be held in the moving frame.

In this letter, we formulate a relativistic GHZ gedanken-experiment with massive particles considering a situation in which measurements are performed by moving observers. It is pointed that GHZ correlations of spins in original directions no longer hold and transfer to different directions in the moving frame. This is a consequence of Wigner rotation [18] and does not imply a breakdown of non-local correlation. To obtain and utilize perfect properties of GHZ correlations again, we should choose spin variables to be measured appropriately to purse desired tasks. Our intention is to explore effects of the relative motion between the sender and receiver which may play role in future relativistic experiment testing the strong conflict between local realism and quantum mechanics, or which may be useful in future quantum information processing using GHZ correlations in high velocity case.

The version of the GHZ experiment in non-relativistic case is listed as follow

[14,19]. Consider three spin-$\frac{1}{2}$ particles prepared in the state

$$|\psi\rangle = \frac{1}{\sqrt{2}}[|\uparrow;\uparrow;\uparrow\rangle + |\downarrow;\downarrow;\downarrow\rangle] \tag{1}$$

Here $|\uparrow\rangle$ represents "up" along the z axis and $|\downarrow\rangle$ signifies "down" along the z axis. Now consider the result of the following products of spin measurements, each made on state $|\psi\rangle$.

(i) Particle 1 along y, particle 2 along y, particle 3 along x. Note that since $\sigma_y\sigma_y\sigma_x|\psi\rangle = -|\psi\rangle$. The product of the yyx measurements should be -1, i.e., the expectation value of three-particle spin correlation in the direction yyx (which is denoted by $E(yyx)$), should be -1.

(ii) Particle 1 along y, particle 2 along x, particle 3 along y. The product of the yxy measurements $E(yxy)$ should be -1.

(iii) Particle 1 along x, particle 2 along y, particle 3 along y. The product of the xyy measurements $E(xyy)$ should be -1.

(iv) Particle 1 along x, particle 2 along x, particle 3 along x. In this case, the product of the xxx measurements $E(xxx)$ should be $+1$.

Obviously spin correlations in directions yyx, yxy, xyy, and xxx are maximally correlated, known as GHZ correlations. And the positive sign in the final scenario is crucial for differentiating between quantum-mechanics and hidden-variable descriptions of reality, because local realistic theory predicts the product be -1. We can see whenever local realism predicts that a specific result definitely occurs for a measurement on one of the particle's spin given the results for the other two, quantum physics definitely predicts the opposite result. Thus, using GHZ correlations, quantum mechanics predictions are in conflict with local realism definitely, while in the case of EPR experiments, quantum mechanics predictions are in conflict with local realism only statisti-

cally. In experiment, GHZ's prediction was confirmed in Ref.[17] using nuclear magnetic resonance.

In relativistic case, the Lorentz transformation induces unitary transformation on vectors in Hilbert space[18]. Suppose that a massive spin-$\frac{1}{2}$ particle moves with the laboratory-frame 4-momentum $p = (m\cosh\xi, m\sinh\xi\sin\theta\cos\phi,$ $m\sinh\xi\sin\theta\sin\phi, m\sinh\xi\cos\theta)$ where the rapidity $\vec{\xi} = \xi\hat{\mathbf{p}}$ with the normal vector $\hat{\mathbf{p}} = (\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta)$. An observer is moving along z-axis with the velocity $\vec{V}$ in the laboratory frame. The rest frame of the observer is obtained by performing a Lorentz transformation $\Lambda = L^{-1}(\vec{\chi}) = L(-\vec{\chi})$ on the laboratory frame with the rapidity $-\vec{\chi} = \chi(-\hat{\mathbf{z}})$. Here $V = \tanh\chi$ and $-\hat{\mathbf{z}} = (0, 0, -1)$ is the normal vector in the boost direction. In this frame, the observer describes the 4-momentum eigenstate $|p\lambda\rangle$ as $U(\Lambda)|p\lambda\rangle$ (for a review of momentum eigenstates, one may refer to Ref.[20,21,22]). A straightforward calculation shows that [8,9,10,11]:

$$U(\Lambda)|p\uparrow\rangle = \cos\frac{\delta}{2}|\Lambda p\uparrow\rangle + e^{i\phi}\sin\frac{\delta}{2}|\Lambda p\downarrow\rangle \tag{2}$$

$$U(\Lambda)|p\downarrow\rangle = -e^{-i\phi}\sin\frac{\delta}{2}|\Lambda p\uparrow\rangle + \cos\frac{\delta}{2}|\Lambda p\downarrow\rangle \tag{3}$$

where $\uparrow$ and $\downarrow$ represent "up" and "down" along z-axis, respectively. The Wigner rotation is indeed a rotation about the direction $\hat{\delta} = (-\hat{\mathbf{z}} \times \hat{\mathbf{p}})/|\hat{\mathbf{z}} \times \hat{\mathbf{p}}|$ through the angle δ:

$$\cos\delta = \frac{A - B(\hat{\mathbf{z}} \cdot \hat{\mathbf{p}}) + C(\hat{\mathbf{z}} \cdot \hat{\mathbf{p}})^2}{D - B(\hat{\mathbf{z}} \cdot \hat{\mathbf{p}})} \tag{4}$$

$$\sin\delta\hat{\delta} = -\frac{B - C(\hat{\mathbf{z}} \cdot \hat{\mathbf{p}})}{D - B(\hat{\mathbf{z}} \cdot \hat{\mathbf{p}})}\hat{\mathbf{z}} \times \hat{\mathbf{p}} \tag{5}$$

with

$$A = \cosh\xi + \cosh\chi \tag{6}$$
$$B = \sinh\xi \sinh\chi \tag{7}$$
$$C = (\cosh\xi - 1)(\cosh\chi - 1) \tag{8}$$
$$D = \cosh\xi \cosh\chi + 1 \tag{9}$$

A GHZ state for three massive particles in the laboratory frame reads:

$$|\psi\rangle = \frac{1}{\sqrt{2}}[|p_1 \uparrow; p_2 \uparrow; p_3 \uparrow\rangle + |p_1 \downarrow; p_2 \downarrow; p_3 \downarrow\rangle] \tag{10}$$

where $p_i = (m\cosh\xi_i, m\sinh\xi_i \sin\theta_i \cos\phi_i, m\sinh\xi_i \sin\theta_i \sin\phi_i, m\sinh\xi_i \cos\theta_i)$ represents the 4-momentum of the i-th particle in the laboratory frame and $i = 1, 2, 3$. Without loss of generality, ϕ_3 is set to 0 which means the third particle moves in the yOz-plane. It is necessary to explicitly specify the motion of the particles because Wigner rotation depends on the momentum. In this GHZ experiment, suppose three spin-$\frac{1}{2}$ particles, prepared in the state (10), move apart from the GHZ source and are detected by three observers. Each observer measures a spin component along a chosen direction. Note that whatever frame is chosen for defining simultaneity, the experimentally observable result is the same [21,23], so we needn't discuss the chronology of spin measurements. Here we assume that, three observers are moving in the z direction at the same velocity $\vec{V}$ in the laboratory frame. What we are interested in are GHZ correlations in the common inertial frame where the observers are all at

rest. In this moving frame, the observers see the GHZ state (10) as:

$$
\begin{aligned}
U(\Lambda)|\psi\rangle = \frac{1}{\sqrt{2}}\Big[&\Big(c_1c_2c_3 - e^{-i(\phi_1+\phi_2)}s_1s_2s_3\Big)|\Lambda p_1\uparrow;\Lambda p_2\uparrow;\Lambda p_3,\uparrow\rangle \\
&+\Big(e^{-i(\phi_1+\phi_2)}s_1s_2c_3 + c_1c_2s_3\Big)|\Lambda p_1\uparrow;\Lambda p_2\uparrow;\Lambda p_3\downarrow\rangle \\
&+\Big(e^{-i\phi_1}s_1c_2s_3 + e^{i\phi_2}c_1s_2c_3\Big)|\Lambda p_1\uparrow;\Lambda p_2\downarrow;\Lambda p_3\uparrow\rangle \\
&+\Big(-e^{-i\phi_1}s_1c_2c_3 + e^{i\phi_2}c_1s_2s_3\Big)|\Lambda p_1\uparrow;\Lambda p_2\downarrow;\Lambda p_3\downarrow\rangle \\
&+\Big(e^{-i\phi_2}c_1s_2s_3 + e^{i\phi_1}s_1c_2c_3\Big)|\Lambda p_1\downarrow;\Lambda p_2\uparrow;\Lambda p_3\uparrow\rangle \\
&+\Big(-e^{-i\phi_2}c_1s_2c_3 + e^{i\phi_1}s_1c_2s_3\Big)|\Lambda p_1\downarrow;\Lambda p_2\uparrow;\Lambda p_3\downarrow\rangle \\
&+\Big(e^{i(\phi_1+\phi_2)}s_1s_2c_3 - c_1c_2s_3\Big)|\Lambda p_1\downarrow;\Lambda p_2\downarrow;\Lambda p_3\uparrow\rangle \\
&+\Big(c_1c_2c_3 + e^{i(\phi_1+\phi_2)}s_1s_2s_3\Big)|\Lambda p_1\downarrow;\Lambda p_2\downarrow;\Lambda p_3\downarrow\rangle\Big]
\end{aligned} \tag{11}
$$

where $c_i \equiv \cos\frac{\delta_i}{2}$, $s_i \equiv \sin\frac{\delta_i}{2}$. And δ_i represents the Wigner angle of the i-th particle which is defined in (4) and (5). Spin operators in relativistic case are defined as [8,9,12]:

$$\sigma_x(p) = |p\uparrow\rangle\langle p\downarrow| + |p\downarrow\rangle\langle p\uparrow| \tag{12}$$
$$\sigma_y(p) = -i|p\uparrow\rangle\langle p\downarrow| + i|p\downarrow\rangle\langle p\uparrow| \tag{13}$$
$$\sigma_z(p) = |p\uparrow\rangle\langle p\uparrow| - |p\downarrow\rangle\langle p\downarrow| \tag{14}$$

Since the observers are moving in the z direction, the directions that are parallel in the laboratory frame remain parallel in the moving frame where the observers are all at rest. However, whether the results of spin measurements in the same direction are still maximally correlated in this moving frame isn't obvious. We now research this question. Let the observer who receives particle 1 performs measurement of σ_y, the observer who receives particle 2 performs measurement of σ_y, and the observer who receives particle 3 performs measurement of σ_x. Thus in the moving frame, the expectation value of three-particle

spin correlation in the direction yyx is obtained:

$$E(yyx) = -\cos\delta_3 \tag{15}$$

Similarly we obtain:

$$E(yxy) = -\cos\delta_2 \tag{16}$$
$$E(xyy) = -\cos\delta_1 \tag{17}$$

We can see that GHZ correlations that are maximally correlated in the laboratory frame no longer appear so in the moving frame. That is, in the moving frame, given the results of measurements on two particles, one can't predict with certainty what the result of a corresponding measurement performed on the third particle. In practice, this means that the relative motion between the source of entangled particles and the observers can alter properties of spin correlations when the observers receive the particles. Thus quantum information processing using these perfect correlations of the GHZ state can't be held, for example, the GHZ experiment can't be done due to lack of knowledge of GHZ correlations in the moving frame.

This effect occurs because Lorentz transformation rotates the direction of spin of the particle as can be seen form (2) and (3). Since the Wigner rotation is in fact a kind of local transformation, it preserves the entanglement of the state [24]. Thus it is reasonable that the GHZ correlation should be preserved in appropriately chosen direction. Here we point out that, to utilize GHZ correlations in the moving frame, the observers should choose spin variables to be measured appropriately according to the wigner rotation:

$$
\begin{aligned}
\sigma_x(\Lambda p_i) \to \sigma_{x'}(\Lambda p_i) &= U(\Lambda)\sigma_x(\Lambda p_i)U(\Lambda)^+ \\
&= (c_i^2 - s_i^2 \cos 2\phi_i)\sigma_x(\Lambda p_i) - s_i^2 \sin 2\phi_i \sigma_y(\Lambda p_i) \\
&\quad -2s_i c_i \cos\phi_i \sigma_z(\Lambda p_i) \qquad (18)\\
\sigma_y(\Lambda p_i) \to \sigma_{y'}(\Lambda p_i) &= U(\Lambda)\sigma_y(\Lambda p_i)U(\Lambda)^+ \\
&= -s_i^2 \sin 2\phi_i \sigma_x(\Lambda p_i) + (c_i^2 + s_i^2 \cos 2\phi_i)\sigma_y(\Lambda p_i) \\
&\quad -2s_i c_i \sin\phi_i \sigma_z(\Lambda p_i) \qquad (19)
\end{aligned}
$$

where the index i represents the i-th particle. Thus GHZ correlations will be obtained in new different directions in the moving frame. For example, if the observer who receives particle 1 measures spin along the direction $y_1' = (-s_1^2 \sin 2\phi_1, c_1^2 + s_1^2 \cos 2\phi_1, -2s_1c_1 \sin\phi_1)$, the observer who receives particle 2 measures spin along the direction $y_2' = (-s_2^2 \sin 2\phi_2, c_2^2 + s_2^2 \cos 2\phi_2, -2s_2c_2 \sin\phi_2)$ and the observer who receives particle 3 measures spin along the direction $x_3' = (c_3^2 - s_3^2, 0, -2s_3c_3)$, maximal correlation $E(y'y'x') = -1$ is obtained again. That is, the GHZ correlation in the direction yyx in the laboratory frame transfers to new direction $y'y'x'$ seen from the moving frame. Similar conclusions are also held for $y'x'y'$ and $x'y'y'$ cases with a careful choice of spin variables according to (18) and (19).

Now we can perform GHZ experiment in the moving frame. After a set of spin measurements along the direction $y'y'x'$, $y'x'y'$, and $x'y'y'$ respectively, local realism will predict the possible outcomes for a $x'x'x'$ spin measurement must be those terms yielding a expectation value $E(x'x'x') = -1$. While quantum theory predicts the outcomes should be the terms yielding $E(x'x'x') = 1$. Then the strong conflict between the quantum theory and the local realism is seen in the moving frame in principle.

Similarly can people test the Bell-type inequality for three-qubit state in relativistic case. In non-relativistic case, any local realistic theory predicts $|\varepsilon| = |E(xyy) + E(yxy) + E(yyx) - E(xxx)| \leq 2$ while the maximal possible

value is reached for the GHZ state where $|\varepsilon| = 4$ [25,26]. If the directions of measurements of spin are fixed as xyy, yxy, yyx, and xxx, the degree of violation for the GHZ state in the moving frame where the observers are at rest equals to:

$$|\varepsilon| = 4\sqrt{(c_1c_2c_3)^4 + (s_1s_2s_3)^4 - 2(c_1c_2c_3s_1s_2s_3)^2\cos\left[2(\phi_1+\phi_2)\right]} \tag{20}$$

The result depends on the velocity of both the particles and the observers with respect to the laboratory in terms of parameters θ, ϕ, and the Wigner angle δ. If the particles (or the observers) are all at rest in the laboratory frame or if the moving direction of the observer is parallel with that of the particle to be measured, the amount of violation reaches to the maximal value $|\varepsilon| = 4$ which gives the same outcome as the case in non-relativistic case. It is interesting to see in some cases the observers will find the degree of violation to be zero. For example, In the case $\xi \to \infty$ and $\chi \to \infty$ and $\theta = \pi/2$, where the particles and the observers move perpendicularly with high velocities, observers will find $|\varepsilon| = 0$. In fact, if three particles rotate angles that are represented by points on the surface $\tan\frac{\delta_1}{2}\tan\frac{\delta_2}{2}\tan\frac{\delta_3}{2} = 1$, the degree of violation for the GHZ state is 0 when $\phi_1 + \phi_2 = n\pi$.

The change in the degree of violation of the Bell-type inequality also results from the fact that the Wigner rotation rotates the direction of spins and thus perfect correlations transfer to different directions as we point out above. As can be seen from (18) and (19), if observers rotate the directions of measurements in accordance with the Wigner rotation, Bell-type inequality turns out to be maximally violated with $|\varepsilon| = 4$.

In summarize, We apply a specific Lorentz boost to the GHZ state and then compute the expectation value of three-particle spin correlations in the trans-

formed state. As a result, spin variables averages that are maximally correlated in the laboratory frame no longer appear so in the same directions seen from the moving frame. The entanglement of the GHZ state is however not lost and perfect correlations of the GHZ state are always possible to be found in different directions seen from the moving frame. As its applications, we formulated GHZ experiment in relativistic case, and Bell-type inequality for three-qubit in relativistic case is also discussed. If the relative motion between the source of entangled particles and the observers must be taken account of, we should consider this GHZ correlation transfer in practice.

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